THE RELATIONSHIP BETWEEN RISK AND EXPECTED RETURNS WITH INCOMPLETE INFORMATION

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Asset pricing theory generally assumes perfect markets and, therefore, asset pricing models disregard the possibility of information deficiency in stock price formation. Our study analyses if the quantity of information about an asset determines its return. More precisely, we want to know if there is a systematic source of information related risk that makes assets which are highly sensitive to this risk factor present higher mean returns. Our results indicate that the market prices the disinformation risk. We find that models which incorporate our attention factor perform better than the traditional CAPM or the Fama and French model, both in time-series analyses and cross-sectionally.

Keywords: Incomplete information, attention by financial analysts and information differential.

(*JEL* G12, G14)

1. Introduction

From its appearance in the literature, the capital asset pricing model (CAPM hereafter) has constituted a fundamental, yet simple, tool in finance to understand the risk-expected return relationship. Its simple formulation makes this model widely accepted; nevertheless its empirical validity has not been fully proven. Over the last 30 years,

numerous empirical anomalies have shown the inability of the model to explain empirical patterns of cross-section of stock returns (Banz, 1981; Basu, 1983; Rosenberg et al., 1985; and most notably Fama and French, 1992).[1]

The CAPM, in common with many other pricing models, implicitly assumes that investors have complete information on the whole stock market. This means that any type of information is instantly diffused and that investors act accordingly and as quickly as possible. Therefore, this assumption makes the asset pricing models ignore the possibility of informational deficiencies and the resulting consequences for risk. However, financial literature suggests that, generally, assets on which the market has relatively less information are considered more risky than those on which more information is available. And, in this way, investors will expect higher returns on the former than on the latter.

There is some literature related to the above argument. The first authors to study the effects of differences in information on the financial markets were Klein and Bawa (1977). They show that without a sufficient level of information, a special (non-traditional) mean-variance method should be applied for the construction of the optimal portfolio. In particular, when there are two subgroups of assets with different levels of information, the authors show that risk-averse investors would limit their attention to the group with more information. Also, Merton (1987) develops a model with incomplete information that gives support to the problem presented here. He considers a market with homogeneous conditional expectations in which investors only receive information on a group of assets, and they only trade with the assets they have information on. Under this incomplete information context, the resulting equilibrium asset pricing model produces an extra mean return, over the market risk premium, which is related to the disinformation cost of the asset for all investors.

Our study is based on the theoretical principles behind the Merton model. We conduct an empirical analysis in order to test if the quantity of information about an asset determines its return. More precisely, we want to know if there is a systematic source of information related risk that makes assets which are highly sensitive to this risk factor present higher mean returns. This is reasonable under the theoretical framework presented by Merton (1987) because investors have homo-

[1] Although this depends on the period analyzed and the frequency of the data.

geneous expectations (there are no information asymmetries), but not all investors have information about all assets. Therefore, there is a lack of information on market as a whole, which allows us to think in terms of a systematic risk factor related to the level of information. Of course, a further question is how information can be measured.

A series of studies by Arbel and Strebel (1982), Arbel *et al.* (1983), Arbel (1985), and Carvell and Strebel (1987) show that stocks with lower attention from financial analysts exhibit higher adjusted returns than stocks with higher attention. These authors identify the attention that analysts give to a firm as a proxy of the level of available information on this firm and they use the number of earnings per share (EPS) estimates to capture and quantify attention. Following these authors, we use the number of earnings estimates by analysts as a proxy of the quantity of information about an asset in the market. It must be noted that this measure captures the total level of information about an asset for the whole set of investors. Thus, this variable must not be interpreted as a proxy of the private information in a stock, as with PIN by Easley, Hvidkjaer and O'Hara (2002).

Our main goal is to translate this individual information for an asset into an aggregate risk factor. For this purpose, we adopt the risk factor construction methodology proposed by Fama and French (1993). In particular, we compute a systematic risk factor as the difference between returns on stocks with low analyst attention and returns on stocks with high analyst attention. In this way, if we are adequately measuring the market premium return for information quantity differences, this attention-based factor will be high in moments when the risk associated to lack of information in the market is high. Therefore, stocks which are positively correlated to this risk factor will be considered by investors as riskier assets and they would present higher mean returns than stocks which are negatively correlated to the attention factor.

Therefore, we analyse the empirical performance of asset pricing models that include the attention-based risk factor in order to determine its contribution to the explanation of returns. Our results indicate that the market prices the disinformation risk. We find that models which incorporate the attention factor perform better than the traditional CAPM or the Fama and French model, both in time-series analyses and cross-sectionally. Moreover, and contrary to the evidence about the seasonal behaviour of the relationship between expected returns

and risk factors based on other firm characteristics, like size or book-to-market ratio, our attention factor contains information about expected returns not only in January. Finally, several robustness analyses confirm the results.

The paper is organized as follows. In Section 2, the theoretical model of Merton (1987) is briefly described. Section 3 accounts for the empirical framework behind this study. Here we describe the proxy used to measure information and the procedure in computing the information risk factor. Section 4 provides the empirical analysis of asset pricing models. This section contains a time-series analysis of the models' performance, a cross-sectional analysis of the risk premiums, a study of the seasonal behaviour of the relationship established by the models, and a robustness subsection. Finally, Section 5 concludes.

2. The model

Merton (1987) develops a capital market equilibrium model with incomplete information. The incompleteness occurs because each investor has information on only a subset of available assets. Within this framework there is no asymmetric information in the sense that all investors with information on an asset have the same information, i.e., they have conditional homogeneous beliefs. But an investor has an incomplete information set if he/she does not know about all available assets.

The derivation of the model in Merton (1987) assumes that the utility of investors is fully determined by the mean and the variance of the wealth at the end of the investing period.[2] However, it can be easily extended to alternative specifications of preferences. Next, we briefly describe the model.

There are N assets and I investors in the economy and we assume a one-factor generating process for the return on assets. The key assumption of the model is that agent i invests in asset j only if he/she has information on asset j. The outcome of this assumption is that the constrained expected utility in a standard optimal portfolio problem for investor i incorporates a term of non information costs:

$$-\sum_{j=1}^{N} \lambda_j^i \omega_j^i, \qquad [1]$$

[2] That is, the assumptions here are the same as in a standard CAPM.

where λ^i_j is the "shadow" cost of asset j for investor i and it is equal to zero if the investor has information on this asset, and ω^i_j is the fraction of initial wealth allocated to asset j by investor i and it is equal to zero if the investor does not have information on this asset.

Solving the optimization problem for the individual investor, it is shown that the shadow cost for asset j is the same for all investors, given that there is no asymmetric information.

$$\lambda^i_j = \Delta_j, i = 1, 2, ..., I \qquad [2]$$

And aggregating among investors, the equilibrium aggregate shadow cost for asset j is,

$$\lambda_j = (1 - q_j) \Delta_j \qquad [3]$$

where q_j is the fraction of all investors with information on j and, of course, it is equal to one if all investors are informed about this asset.

Finally, aggregating among securities, we can obtain the market shadow cost of incomplete information:

$$\lambda_m = \sum_{j=1}^{N} x_j \lambda_j \qquad [4]$$

x_j is the fraction of the market portfolio invested in security j. From the equilibrium condition, under this incomplete information framework, it can be shown that

$$x_j = \frac{q_j \Delta_j}{\delta \sigma_j^2} \qquad [5]$$

where δ is the risk aversion coefficient, and σ_j^2 is the variance of the return of the asset j.

The resulting equilibrium asset pricing model is:

$$E(R_j) = R_f + \beta_j (E(R_m) - R_f) + \lambda_j - \beta_j \lambda_m. \qquad [6]$$

Equation [6] states that, with incomplete information, the expected return of an asset is not only a function of the market risk premium, as in a standard CAPM, but it also contains a disinformation premium such that the higher the shadow cost of the asset, the higher its expected return. Moreover, this additional premium does not depend exclusively on the shadow cost of the asset but it also depends on the

relation between the shadow cost of the asset and the shadow cost
of the full market. In the case that the market beta of the asset is
equal to one and the level of disinformation of this asset is equal to
the aggregate disinformation in the whole market, this premium will
be zero. Otherwise, this additional term can be positive or negative
depending on the magnitude of λ_j in relation to the magnitude of
$\beta_j \lambda_m$. In this way, the model establishes that investors demand high
returns for those assets which, although they present low market risk,
have little information on them; in the sense that the investors with
information on these assets are a small fraction of the total investors
in the market.[3]

3. The empirical framework

The aim of this study is to test the role of the degree of information
in determining stock returns in line with Merton's (1987) model. To
investigate this issue, we need to make some simplifying assumptions
to the theoretical specification in equation [6]. First, we have to ap-
proximate the unobservable variables.

An equally-weighted index of the stocks listed at each moment in time
is used as a proxy of the market portfolio, and we use the monthly
equivalent of the annual interest rate on Treasury Bills as the risk free
rate.

A more difficult task is to approximate the shadow cost for each asset
and the corresponding aggregate cost.[4]

Our empirical investigation is motivated by the predictions of the Mer-
ton model, but not constitute a direct test of that model.

3.1   A proxy for the level of information and its relation with return

The key variable in this incomplete information model is the shadow
price of disinformation about the firm. To empirically test the perfor-
mance of equation [6], we need to approximate this variable. Instead
of this, our purpose is to measure the degree of information that the
set of investors in the market have about each firm. In line with Arbel
and Strebel (1982), and Carvell and Strebel (1987), we use the number
of professional analysts following the firm as a measure of the quan-

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[3] Of course, the model can also be interpreted as a violation of the efficiency of the
market portfolio.

[4] Probably, this is the reason why asset pricing models under incomplete information
have not received so much attention in empirical studies.

tity of information that investors have on a firm. It must be noted that we want to use the incompleteness but not the asymmetry in the information set; in this last case the dispersion on the earnings estimates would be a better proxy. Given equation [5], the shadow cost is inversely proportional to the degree of investor recognition (q_j),

$$\lambda_j = \frac{x_j \delta \sigma_j^2}{q_j}.$$ [7]

Therefore, if the number of analysts providing earning estimates on a firm (attention level) is small (q_j is high), we can infer that there is little available information on this firm (the shadow cost λ_j is high) and, from equation [6], we expect its asset return to be higher than the prediction of a standard CAPM.

The monthly number of EPS estimates for the current year provided by the I/B/E/S (Institutional Broker Estimation System) and Facset-JCF database has been used. In cases where this information is not available, we consider the number of estimates to be zero.

To give insights into the empirical consistency of the model [6], we analyse some statistics of portfolios based on attention level. For stocks traded on the Spanish stock market during the period December 1990 to December 2006 we compute monthly returns, adjusted for dividends, seasoned equity offerings and splits. We rank stocks according to the number of earnings estimates each month, and we group them into five portfolios. Thus, portfolio $A1$ reflects the lowest level of attention and portfolio $A5$ the highest. After this, we compute each portfolio return, in the next month, equally weighting each stock return in the portfolio.[5]

Table 1 provides the mean and the standard deviation of the portfolio returns, the mean number of stocks in each portfolio, the mean number of analysts following the firms in each portfolio, the mean size and the mean book-to-market ratio of the portfolios. These statistics are computed as the average of the corresponding variables for the

[5]The sorting criterion does not give us a wide range of portfolios for two reasons. Firstly the variable used (attention level) is not continuous. This means that it is not possible to homogenously distribute the number of stocks into the portfolios. Thus, the number of portfolios is small to ensure that all portfolios have a sufficiently large number of stocks. Secondly, the monthly frequency of the data means the attention variable have some missing values for some stocks. This reduces the number of available stocks within each month.

individual stocks in each portfolio. Also, the market betas and the Jensen's alphas from the CAPM for each portfolio are provided. The numbers in parenthesis below the alphas are the p-values of individual significance. Finally, in the last row, a Chi-square test for equality in alphas between portfolios $A1$ and $A5$ is computed.

TABLE 1
Attention portfolios. Summary Statistics

	Mean Return	Standard Deviation	Number Stocks	Attention	Size	Book-to Market	Beta	Alpha
A1	0.0166	0.0676	14.05	0.32	1,398.19	1.09	0.99	0.0018 (0.45)
A2	0.0139	0.0651	17.95	4.54	191.92	1.29	1.03	-0.0012 (0.47)
A3	0.0152	0.0662	19.15	12.98	680.23	0.95	1.07	-0.0004 (0.78)
A4	0.0149	0.0652	19.59	25.91	1,327.98	0.89	1.04	-0.0004 (0.79)
A5	0.0141	0.0572	18.32	40.75	6,154.23	0.88	0.86	0.0007 (0.71)
A1-A5								0.0011
χ^2								0.137
p-value								(0.71)

Notes:

1. The table is based on all sample observations (firm-monthly) during the period January 1991-December 2006 where we have data about returns, market value, book-to-market, and number of financial analysts' estimates. This table provides average monthly values.
2. Portfolios are quintiles based on the level of attention. *A1* is the portfolio with the lowest level of attention and *A5* is the portfolio with the highest level of attention.
3. Number Stocks: Mean number of stocks within each portfolio; Attention: Mean number of analysts following firms within each portfolio; Size: Mean market value in MM € of each portfolio. Book-to-Market: Mean Book-to-Market of each portfolio.
4. Beta and Alpha are calculated from the CAPM. Numbers in parenthesis are p-values for individual significance.
5. *A1-A5* is the portfolio long in the lowest attention assets and short in the highest ones. χ^2 is the Chi-square statistic for equality in alphas.

Table 1 shows that there is not much dispersion in mean returns among the five portfolios, although portfolio $A1$ presents the highest mean return and the highest volatility. In order to check the implications of the model [6] by the data, we use the alpha values. In theory, we might observe decreasing alphas from portfolio $A1$ to portfolio $A5$, but this is not the case. Although again portfolio $A1$ presents the highest alpha, it is not possible to reject the null of alpha1 equal to alpha5, as can be seen in this table.

Assuming the theoretical model, alphas here represent the disinformation premium:

$$\alpha_j = \lambda_j - \beta_j \lambda_m$$

Thus, the magnitude of the alpha portfolio depends on its beta. As Table 1 shows, higher betas are associated with lower alphas, and vice versa, according to the prediction of the model.

But, why are alphas not decreasing from the portfolio with the lowest degree of information to the portfolio with the highest degree of information? Coming back to equation [5], the shadow cost of an asset, and thus its alpha, depends not only on its information degree but also on its size and its volatility. More specifically, among firms with the same market risk, firms with larger specific variance will have larger alphas. This is consistent with empirical findings that expected returns depend on both market risk and idiosyncratic risk (Campbell, *et al.,* 2001; Ang, *et al.,* 2006, among others). Moreover, among firms with the same volatility and the same degree of investor recognition, larger firms will have larger alphas. The combination of these different effects makes it difficult to find a structure in the alphas of the five attention portfolios. As can be seen in Table 1, the five portfolios present similar volatility, but their sizes are very different.

On the other hand, as financial literature suggests, there is a strong relationship between the size and the number of analysts following the firm. As Bhushan (1989) suggests, the level of analyst coverage received by a firm is determined by several firm characteristics and size is one of them. Moreover, Chen *et al.* (2002) show that there is a positive relationship between analyst following and market value. The explanation is related to the decrease in the agency costs as a consequence of the analysts' role in the markets (Jensen and Meckling, 1976). For these reasons, in order to avoid possible interactions, we follow a proposal presented by Hong et al. (2000) that controls for the effect of firm size on its level of analyst coverage. Therefore, as a second proxy for the level of information we use the residual of the coverage level on firm size.

$$\log\left(1 + EST_{jt}\right) = \beta_0 + \beta_1 \log\left(SIZE_{jt}\right) + \varepsilon_{jt} \qquad [8]$$

where EST_{jt} represents the number of annual earnings estimates for firm j in month t, $SIZE_{jt}$ represents the size of firm j in month t, and the estimate of ε_{jt} will represent the level of residual attention.

Again, we rank stocks according to residual attention and this time place them in ten portfolios $(RA1, \ldots, RA10)$.[6] Table 2 reports the results. We can not observe strictly decreasing alphas from $RA1$ to $RA10$, but alphas are negative in four out of the five portfolios with the highest residual attention and alphas are positive in three out of the five portfolios with the lowest residual attention. Moreover, the null of equality in alphas for portfolio $RA1$ and portfolio $RA10$ is now strongly rejected, given an extra return of 1.2% for the portfolio with the lowest attention level. As before, there is an inverse relationship between alphas and market betas.

TABLE 2
Residual attention portfolios. Summary Statistics

	Mean Return	Standard Deviation	Number Stocks	Residual Attention	Size	Book-to Market	Beta	Alpha
RA1	0.0191	0.0681	6.32	-2.44	3,085.12	0.79	0.85	**0.0057**
								(0.09)
RA2	0.0117	0.0742	7.46	-1.20	396.86	1.30	1.02	-0.0033
								(0.29)
RA3	0.0140	0.0658	8.86	-0.51	1,590.72	1.33	0.95	-0.0004
								(0.88)
RA4	0.0177	0.0609	9.41	-0.07	2,583.06	1.11	0.88	**0.0040**
								(0.07)
RA5	0.0179	0.0626	9.21	0.23	3,710.57	0.82	0.95	**0.0035**
								(0.07)
RA6	0.0149	0.0680	9.36	0.44	3,168.67	0.79	1.05	-0.0004
								(0.83)
RA7	0.0157	0.0636	9.79	0.60	2,444.81	0.79	0.94	0.0015
								(0.51)
RA8	0.0127	0.0680	9.75	0.74	1,352.07	0.83	1.02	-0.0023
								(0.29)
RA9	0.0151	0.0694	9.82	0.91	703.92	0.88	1.07	-0.0005
								(0.82)
RA10	0.0107	0.0767	9.06	1.21	328.20	0.84	1.17	**-0.0058**
								(0.02)
RA1-RA10								**0.0115**
χ^2								7.91
p-value								(0.005)

Notes:
1. The table is based on all sample observations (firm-monthly) during the period January 1991-December 2006 where we have data about returns, market value, book-to-market, and number of financial analysts' estimates. This table provides average monthly values.
2. Portfolios are deciles based on the residual attention extracted from equation (7). *RA1* is the portfolio with the lowest level of residual attention and *RA10* is the portfolio with the highest level of residual attention.
3. Number Stocks: Mean number of stocks within each portfolio; Residual Attention: Mean residual attention; Size: Mean market value in MM €. Book-to-Market: Mean Book-to-Market.
4. Betas and Alphas are calculated from CAPM. Numbers in parenthesis are p-values for individual significance.
5. *RA1-RA10* is the portfolio long in the lowest residual attention assets and short in the highest ones. χ^2 is the Chi-square statistic for equality in alphas.

[6] The definition of this proxy of information degree allows us to have a bigger number of portfolios and a similar number of assets in each portfolio.

These results suggest that a relationship between returns and level of information exists. Also the results indicate that we are reasonably approximating the level of information on an asset by the number of analysts following the firm because the most followed stocks present lower returns than the least followed stocks. Moreover, this extra return persists when controlling for market risk. Therefore, to propose a risk factor based on this variable and to test its contribution within asset pricing models makes sense.

3.2 The disinformation risk premium

The equilibrium asset pricing model in equation [6] exhibits an additional risk premium to the CAPM. This disinformation premium in expected returns depends on the relationship between the shadow cost of the firm and the whole shadow cost in the market.

$$E\left(R_j\right) = R_f + \beta_j \left(E\left(R_m\right) - R_f\right) + \lambda_j - \beta_j \lambda_m. \qquad [9]$$

As we said before, measuring the shadow costs is not an obvious task, especially for the whole market. For this reason, we propose the computation of an information risk factor which we expect to be determinant in explaining mean returns under an arbitrage pricing framework. This risk factor can be easily computed and, in our understanding, the associated premium return retains the interpretation of the disinformation premium shown in the theoretical model [6].

Our purpose is to construct an aggregate risk factor, following the methodology of Fama and French (1993) and based on the proxy of information computed in the section above. Our aim is to test the capability of this new risk factor in explaining both time series and cross-sectional returns against the successfully recognized size and book-to-market factors by Fama and French. Now we describe the construction procedure for all the factors used in this study.

First we compute the three factors of Fama and French (1993): the excess market return, the size factor (SMB) and the book-to-market factor (HML).[7] The size and the book-to-market of each firm come from CNMV and Global Compustat.

Our information risk factor is a differential of returns of assets with low residual attention and assets with high residual attention. The

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[7] See Fama and French (1993) for a detailed description.

use of this proxy for the level of information instead of directly using attention levels is due to the strong correlation found between firm size and the number of financial analysts following the firm. This allows us to have a factor implicitly uncorrelated to SMB. Given that we also find some relationship between residual attention levels and book-to-market, as shown in Tables 1 and 2, we make these two characteristics interact in constructing the information factor. Firstly, we sort stocks into three book-to-market groups (low (L), medium (M) and high (H)), and into three levels of residual attention groups (low attention (LA), medium attention (MA) and high attention (HA)). Later, we form nine portfolios of interactions between the three book-to-market and the three residual attention groups: L/LA, L/MA, L/HA, M/LA, M/MA, M/HA, H/LA, H/MA and H/HA. Finally, we obtain the risk factor associated with residual attention ($LMMA$) as the difference, each month, between the simple average of the three lowest attention levels portfolios (L/LA, M/LA, H/LA) and the simple average of the three highest attention levels portfolios (L/HA, M/HA, H/HA).

TABLE 3
Fama-french and attention risk factors

Panel A: Summary Statistics						
	Mean	Maximun	Minimun	Standard Deviation	Skewness	Kurtosis
$R_m - R_f$	0.0101	0.2101	-0.1688	0.0597	0.3211	4.1139
SMB	-0.0025	0.1230	-0.1436	0.0404	0.3519	4.0054
HML	0.0008	0.1916	-0.1097	0.0397	0.6156	5.8557
$LMMA$	0.0032	0.3029	-0.0847	0.0466	1.8198	11.8355
Panel B: Correlations between Risk Factors						
	$R_m - R_f$		SMB		HML	$LMMA$
$R_m - R_f$	1					
SMB	0.37		1			
HML	0.01		-0.32		1	
$LMMA$	-0.15		0.20		-0.18	1

Notes:
1. The table is based on all sample observations (firm-monthly) during the period January 1991-December 2006 where we have data about stocks returns, market value, book-to-market, and number of financial analysts' estimates.
2. Panel A provides summary statistics. $R_m - R_f$, SMB and HML are the three Fama-French factors and $LMMA$ is our attention-based risk factor.
3. Panel B reports correlations between the four factors.

To verify that the risk factor construction process has been adequate, we test for the degree of correlation between them. In Table 3 we present the descriptive statistics of the four risk factors. As can be seen, SMB presents a negative mean, corroborating the disappear-

ing of the size anomaly since the nineties. As expected, $LMMA$ have a positive mean, indicating that low attention stocks offer higher mean returns than high attention stocks. The correlations between the SMB, HML, and $LMMA$ are within reasonable levels, and the correlations between the first two factors and the market are similar to those found in the literature.[8] The negative correlation between the attention-based factor and the market might not be surprising; as LMMA represents a measure of aggregate risk, it is high in stressed periods in which returns are low.

4. Testing the model

In this section, we analyse the performance of asset pricing models which incorporate the three Fama-French factors and our attention-based factor. The study will be done for both time series and cross-section of mean returns. The set of assets to be explained by the models analysed here could have been the ten residual attention portfolios described in Section 3. However, Lewellen *et al.* (2007) show that for returns with a covariance structure similar to the risks factors, loadings on any proposed factor will line up with the true expected returns as long as the factor weakly correlates with the common source of variation in returns. For this reason, using portfolios constructed on the same basis as the risk factor may generate high cross-sectional R^2 even though this factor is not able to explain the cross-section of true expected returns. The problem could be solved by expanding the set of testing assets to include portfolios that break down the specific covariance structure of the portfolio returns. Thus, we use twenty portfolios resulting from sorting stocks by four criteria: five attention-based portfolios, five size-based portfolios, five market beta-based portfolios, and five book-to-market-based portfolios.

4.1   Time series analysis

We now analyse various specifications of the relationship between expected returns and risk using the following equation.

$$E\left(R_p\right) = r_f + \left(E\left(R_m\right) - r_f\right)\beta_{mp} + E\left(SMB\right)\beta_{SMBp} +$$
$$+ E\left(HML\right)\beta_{HMLp} + E\left(LMMA\right)\beta_{LMMAp} \qquad [10]$$

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[8] Fama and French (1993) in the US market and Nieto (2004) in the Spanish market obtain similar correlation levels.

To do so, we estimate different versions of the following regression for each portfolio in our sample.

$$R_{pt} - r_{ft} = \alpha_p + (R_{mt} - r_{ft})\,\beta_{mp} + SMB_t\beta_{SMBp} +$$
$$+HML_t\beta_{HMLp} + LMMA_t\beta_{LMMAp} + \pi_{pt}, \quad p = 1, 2, ..., 20 \qquad [11]$$

Table 4 provides some statistics related to the fit of each model considered. The first two columns report statistics for the test of alphas being jointly equal to zero. If the model is correctly specified, the intercepts in [11] will be zero for all portfolios jointly.

W denotes a Wald statistic of the null hypothesis.

$$W = T\left(\widehat{\alpha}'\widehat{\Sigma}^{-1}\widehat{\alpha}\right) \qquad [12]$$

Where $\widehat{\alpha}$ is a (20x1) vector of the estimated intercepts, and $\widehat{\Sigma}$ is residual covariance matrix from regression [11]. Since the covariance matrix is unknown, under the null hypothesis W has an asymptotic chi-square distribution with degrees of freedom equal to the number of restrictions (20).

GRS is the Gibbons, Ross, and Shanken (1989) statistic. The authors adjust the Wald statistic to take into account the fact that the sample proxy for the representative portfolio may not be efficient. In this case, we use its finite-sample distribution which is a central F.

$$GRS = \left(\frac{T - N - k}{NT}\right)\left(1 + f'S^{-1}f\right)^{-1}W \; [] \; F\left(N, T - N - k\right) \quad [13]$$

Where f is a k-vector of the mean of the factors in the model, S is the covariance matrix of the factors, T is the number of observations (192 months), N is the number of portfolios (20), and k is the number of risk factors considered in the model.

The last two columns report the average R^2 and the average adjusted R^2 between the twenty estimated regressions for each model.

TABLE 4
Time series estimation

	Wald	GRS	R^2 (%)	Adj. R^2 (%)
MODEL 1	40.36	1.75	84.70	84.62
	0.00	0.03		
MODEL 2	37.48	**1.60**	85.56	85.41
	0.01	0.06		
MODEL 3	37.97	**1.59**	88.69	88.51
	0.01	0.06		
MODEL 4	**23.60**	0.97	87.90	87.70
	0.26	0.50		
MODEL 5	41.60	1.76	86.20	85.98
	0.00	0.03		
MODEL 6	**25.62**	1.05	89.34	89.11
	0.18	0.41		

Notes:
1. The table is based on all sample observations (firm-monthly) during the period January 1991-December 2006 where we have data about stocks returns, market values, book-to-market ratios, and number of financial analysts' estimates.
2. This table reports performance measures of the following time series regressions for twenty portfolios constructed by sorting stocks into five attention portfolios, five size portfolios, five boot-to-market portfolios, and five market beta portfolios.

$$\text{MODEL 1: } R_{pt} - r_{ft} = \alpha_p + (R_{pt} - r_{ft})\,\beta_{mp} + \upsilon_{pt}$$
$$\text{MODEL 2: } R_{pt} - r_{ft} = \alpha_p + (R_{pt} - r_{ft})\,\beta_{mp} + LMMA_t\,\beta_{LMMAp} + z_{pt}$$
$$\text{MODEL 3: } R_{pt} - r_{ft} = \alpha_p + (R_{pt} - r_{ft})\,\beta_{mp} + SMB_t\,\beta_{SMBp} + HML_t\,\beta_{HMLp} + \upsilon_{pt}$$
$$\text{MODEL 4: } R_{pt} - r_{ft} = \alpha_p + (R_{pt} - r_{ft})\,\beta_{mp} + SMB_t\,\beta_{SMBp} + LMMA_t\,\beta_{LMMAp} + \varepsilon_{pt}$$
$$\text{MODEL 5: } R_{pt} - r_{ft} = \alpha_p + (R_{pt} - r_{ft})\,\beta_{mp} + HML_t\,\beta_{HMLp} + LMMA_t\,\beta_{LMMAp} + \eta_{pt}$$
$$\text{MODEL 6: } R_{pt} - r_{ft} = \alpha_p + (R_{pt} - r_{ft})\,\beta_{mp} + SMB_t\,\beta_{SMBp} + HML_t\,\beta_{HMLp} + LMMA_t\,\beta_{LMMAp} + \pi_{pt}$$

Columns two and three are statistics for the null hypothesis of alphas jointly equal to zero. Wald: The statistic of Wald test and its p-value. GRS: Gibbons, Ross, and Shanken (1989) statistic and its p-value. Columns four and five show the average of the R^2 and adjusted R^2 between the twenty portfolios for each model.

The main conclusion from the results in Table 4 is that the attention factor must be considered to improve the performance of standard asset pricing models such as the CAPM or the Fama and French model. With regard to the test of alphas, with the exception of the model that includes the market factor, HML, and $LMMA$, the GRS statistic can not reject the null in all regressions in which the attention factor is included. In fact, while its p-value is near to the usual 5 percent level of rejection for the Fama and French model, when the attention factor is added (model 6) the p-value stands at 41 percent, far from the rejection area. Also the R^2 and the adjusted R^2 increase when the

$LMMA$ factor is added to both the standard CAPM and the Fama-French model.

4.2 Cross-sectional analysis

The time series analysis above seems to indicate that models which include our attention factor behave better. However, we also want to know whether the risk incorporated into each factor considered is priced by investors when they require a specific return. In other words, we want to know whether the risk premium associated with the factors are relevant in determining the cross-section expected returns. To test this we carry out the Fama and MacBeth (1973) estimation and test procedure.

The test has two stages: in the first we estimate the betas of the excess returns on the twenty portfolios with respect to the factors in a rolling time series regression. In the second stage we estimate the risk premiums of these betas with a cross-sectional regression.

The first beta is estimated with a set of 36 observations, corresponding to our first three sample years and it is assigned to December 1993 for the subsequent cross-section estimation. Next, the betas corresponding to January 1994 are estimated with the first 37 observations. We continue successively up to a total of 60 months. From here the set of observations for the beta estimation remains constant, incorporating an additional observation as it eliminates the first one. In the end, we have a series of 157 observations (December 1993-December 2006) for the cross-sectional analysis.

After estimating the explanatory variables, we carry out the following cross-sectional estimation for each of the 157 months.

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_{mpt} + \gamma_{2t}\widehat{\beta}_{SMBpt} + \gamma_{3t}\widehat{\beta}_{HMLpt} + \gamma_{4t}\widehat{\beta}_{LMMApt} +$$
$$+ u_{pt}, \quad p = 1, 2, ..., 20 \qquad [14]$$

The final estimator is the average of the 157 monthly estimates of gammas and its t-statistic for individual significance is calculated using the variance of the series divided by 156. Also, the Shanken (1992) t-statistic, which adjusts the variance of the estimates for measurement errors in unobservable variables, is computed.

The results of the risk premium estimates in the six models are shown in Table 5. In each column we present the risk premium of each beta

considered by the model, the standard t-statistic in parenthesis, and the Shanken adjusted t-statistic in brackets. In the last two columns we present two performance measures. First, the adjusted R-square coefficient is provided in percentage. It is obtained from the sum of the 157 total sums and the 157 residual sums of each regression. The sum of the twenty mean square residuals (SSR) for each analysed model is provided in the last column in 10,000 percent numbers.

TABLE 5
Cross-sectional regressions

γ_0	γ_1	γ_2	γ_3	γ_4	Adj. R^2 (%)	SSR
0.0144	0.0020				24.63	0.676
(4.58)	(0.41)					
[4.57]	[0.41]					
0.0130	0.0034			**0.0075**	42.64	0.569
(4.12)	(0.67)			(1.68)		
[4.06]	[0.66]			[1.65]		
0.0142	0.0022	0.0026	-0.0016		48.56	0.527
(4.08)	(0.44)	(0.76)	(-0.43)			
[4.07]	[0.44]	[0.76]	[-0.43]			
0.0105	0.0059	0.0001		**0.0090**	49.55	0.554
(3.02)	(1.08)	(0.03)		(2.01)		
[2.93]	[1.05]	[0.03]		[1.95]		
0.0124	0.0040		0.0002	0.0066	48.15	0.483
(3.91)	(0.80)		(0.05)	(1.53)		
[3.85]	[0.79]		[0.05]	[1.51]		
0.0098	0.0067	0.0001	0.0016	**0.0077**	52.67	0.473
(2.85)	(1.24)	(0.05)	(0.45)	(1.76)		
[2.77]	[1.20]	[0.04]	[0.44]	[1.71]		

Notes:
1. The table is based on all sample observations (firm-monthly) during the period January 1991-December 2006 where we have data about stocks returns, market values, book-to-market ratios, and number of financial analysts' estimates.
2. This table reports the estimates from the two-step cross-sectional Fama-MacBeth procedure for the following regression, from 1993:12 to 2006:12.

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{mpt} + \gamma_{2t}\hat{\beta}_{SMBpt} + \gamma_{3t}\hat{\beta}_{HMLpt} + \gamma_{4t}\hat{\beta}_{LMMApt} + \upsilon_{pt}$$

The dependent variable is the monthly return on twenty portfolios constructed by sorting stocks into five attention portfolios, five size portfolios, five boot-to-market portfolios, and five market beta portfolios. The explanatory variables are the betas of the different factors, and they are estimated with previous data to each cross-sectional estimation, including the month in question, by the time-series regression.

$$R_{pt} - R_{ft} = \alpha_p + \beta_{pm} (R_{mt} - R_{ft}) + \beta_{SMBp} SMB_t + \beta_{HMLp} HML_t + \beta_{LMMApt} LMMA_t + \varepsilon_{pt}$$

($R_{mt} - R_{ft}$), SMB and HML are the three Fama-French factors and LMMA is a risk factor constructed as the difference between the return of the less attended stocks minus the return of the more attended ones.
The risk premium estimates are the average of the coefficient estimates from the monthly cross-sectional regressions. t-values for individual significance, calculated using the standard deviation of the series of estimates, are reported in parenthesis. The corresponding adjusted t-statistics by Shanken (1992) are in brackets.
3. *Adj. R^2* is the adjusted R-square computed from the R-square obtained with the sum of all total sums and all residual sums from the monthly cross-sectional regressions. *SSR* is the sum of the square of the residuals for the twenty portfolios.

Results in Table 5 show that neither the market beta, nor SMB nor HML betas present a significant price in terms of returns in the Spanish market.[9] However, the risk associated to our attention-based factor seems to contain information about the differences in cross-sectional returns. Both statistics (standard and Shanken) of individual significance reject the null of an attention risk premium equal to zero, whatever the model analysed, although only at ten percent of significance level. However, both performance measures (R^2 and SSR) strongly confirm the contribution of our factor to the goodness of fit of the model. As an example, the adjusted R^2 goes from 24.63 percent to 42.64 percent (from 48.56 percent to 52.67 percent) when $LMMA$ is added to a standard CAPM (Fama-French model). In the same line, the square residuals of the models are reduced when $LMMA$ is included, going from 0.676 to 0.569 in the case of the CAPM, or from 0.527 to 0.473 in the case of the Fama-French model.

In order to thoroughly investigate the gain of including the $LMMA$ factor in explaining the returns on the twenty portfolios, Table 6 reports the average residuals from equation [14] and for each portfolio. The first column covers the CAPM, the residuals from the Fama-French model are in the second column, and the third column contains the residuals from the four factor model. These residuals represent the return not explained by the model and they are in percent numbers. As can be seen, the CAPM fails to explain a considerable part of the return on several of the twenty portfolios. The pricing errors are big for the two extreme attention portfolios (0.24 for $A1$ and -0.15 for $A5$), for the portfolio with highest market beta assets (-0.23), for the fourth size portfolio (0.23), and for the medium book-to-market portfolios (0.19, -0.52, and 0.22 for $BTM2$, $BTM3$, and $BTM4$, respectively). The Fama-French model reduces the pricing errors for all the portfolios with respect to the CAPM, in general terms. However, the returns on some portfolios are still badly explained by the model; the most striking cases are for $Size4$ and $BTM3$. When the $LMMA$ is added to the three Fama-French factors, with the exception of portfolio $A1$, the model is able to better adjust the returns on these conflicting portfolios than the CAPM and the Fama-French model.

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[9] These results confirm previous evidence about the performance of the CAPM or the Fama-French model with Spanish Market data. See Nieto (2004).

TABLE 6
Pricing errors

	CAPM	FF	4-FACTORS
A1	0.2357	0.1343	-0.1728
A2	-0.0543	-0.0676	-0.0197
A3	0.0716	0.0084	0.1033
A4	-0.0961	0.0254	0.0494
A5	-0.1472	-0.0908	-0.0355
Beta1	0.0371	-0.0410	0.0487
Beta2	-0.0263	-0.0470	-0.0408
Beta3	0.1375	0.0678	0.0518
Beta4	0.1108	0.1798	0.1434
Beta5	-0.2276	-0.1599	-0.1173
Size1	-0.1238	-0.1602	-0.1769
Size2	0.1520	0.0580	0.1743
Size3	-0.1797	-0.1500	-0.1026
Size4	0.2335	0.2768	0.1734
Size5	-0.1082	-0.0420	-0.0899
BTM1	0.0066	0.0900	0.1362
BTM2	0.1887	0.1867	0.1869
BTM3	-0.5167	-0.4822	-0.4590
BTM4	0.2236	0.1278	0.0412
BTM5	0.0830	0.0858	0.1058

Notes:
1. The table is based on all sample observations (firm-monthly) during the period January 1991-December 2006 where we have data about stocks returns, market values, book-to-market ratios, and number of financial analysts' estimates.
2. This table reports the average portfolio residuals from three versions of the following monthly cross-sectional regressions, from 1993:12 to 2006:12.

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{mpt} + \gamma_{2t}\hat{\beta}_{SMBpt} + \gamma_{3t}\hat{\beta}_{HMLpt} + \gamma_{4t}\hat{\beta}_{LMMApt} + \upsilon_{pt}$$

CAPM is the model which only considers market risk, FF is the Fama-French model which considers market risk, size risk and book-to-market risk, and 4-FACTORS is a model that adds an attention risk factor to the Fama-French model. The dependent variable is the monthly return on twenty portfolios constructed by sorting stocks into five attention portfolios (A1-A5), five market beta portfolios (Beta1-Beta5), five size portfolios (Size1-Size5), and five boot-to-market portfolios (BTM1-BTM5). The explanatory variables are the betas with respect to market ($R_{mt} - R_{ft}$), to the Fama-French factors (SMB and HML), and to a risk factor constructed as the difference between the return of the less attended stocks minus the return of the more attended ones (LMMA). These betas are estimated with previous data to each cross-sectional estimation.

Summarizing, although the *t*-statistics for the null of individual insignificance of the attention-factor are not so convincing given that the rejection occurs at ten or more percent level, the rest of the analysis in this Section confirms the relevance of including a risk factor

related to the degree of information in the whole market under an incomplete information asset pricing model. The global performance measures indicate that the model fits better than standard models under complete information, both in time-series analysis (GRS statistic) and cross-sectionally (adjusted R^2 and SSR), and also the model explains the returns on the different portfolios more accurately.

4.3   Seasonal behaviour

Empirical evidence has shown that stock returns present a seasonal behaviour. In particular, returns in January are higher than for the rest of the year. Empirical findings show that the January anomaly is related to some firm characteristics, with returns being higher in January for small firms or firms with low stock prices, as examples.[10]

TABLE 7
Portfolio returns. Summary statistics: January Effect

	A1	A2	A3	A4	A5	Beta1	Beta2	Beta3	Beta4	Beta5
M Ja.	0.0682	0.0773	0.0688	0.0534	0.0362	0.0424	0.0531	0.0480	0.0762	0.0801
SD Ja.	0.0786	0.0881	0.0855	0.0709	0.0623	0.0565	0.0692	0.0690	0.0754	0.1071
M n-Ja.	0.0132	0.0107	0.0129	0.0124	0.0126	0.0130	0.0120	0.0149	0.0122	0.0105
SD n-Ja.	0.0613	0.0528	0.0553	0.0551	0.0535	0.0360	0.0419	0.0533	0.0632	0.0774
	Size1	Size2	Size3	Size4	Size5	BTM1	BTM2	BTM3	BTM4	BTM5
M Ja.	0.0945	0.0759	0.0588	0.0460	0.0294	0.0499	0.0560	0.0514	0.0764	0.0688
SD Ja.	0.1047	0.0885	0.0750	0.0622	0.0599	0.0651	0.0800	0.0663	0.0817	0.0949
M n-Ja.	0.0092	0.0123	0.0110	0.0164	0.0130	0.0128	0.0149	0.0079	0.0138	0.0125
SD n-Ja.	0.0639	0.0569	0.0535	0.0532	0.0500	0.0617	0.0526	0.0499	0.0525	0.0581

Notes:

1. The table is based on all sample observations (firm-monthly) during the period December 1993-December 2006 where we have data about stocks returns, market values, book-to-market ratios, and number of financial analysts' estimates.

2. This table reports the mean and the standard deviation of the return on twenty portfolios constructed by sorting stocks into five attention portfolios *(A1-A5)*, five market beta portfolios *(Beta1-Beta5)*, five size portfolios *(Size1-Size5)*, and five boot-to-market portfolios *(BTM1-BTM5)*.

3. The sample is divided into January months and the rest of the year. *M Ja.* and *SD Ja.* refer to the mean and the standard deviation, respectively, of the return on each portfolio employing only January months in our sample. *M n-Ja.* and *SD n-Ja.* refer to the mean and the standard deviation of the return on each portfolio excluding January months from our sample.

This January effect is also found in our sample data. Table 7 provides the mean and the standard deviation of the returns on our twenty portfolios separating the data into January months and non-January

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[10]The January anomaly has been documented in many studies including Banz (1981), Blume and Stambaugh (1983), Keim (1983), Jaffe *et al.* (1989) and Bhardwaj and Brooks (1992).

months. For all of the portfolios the mean return is substantially larger in January than for the rest of the year. Moreover, the January anomaly is also related to some firm characteristics. Table 7 reports that there is a strong relationship between the extra January return and the four characteristics used here to sort stocks into portfolios. In particular, the differential return between January and the rest of the months is higher for the lowest attention stocks ($A1$), for stocks with the highest market betas ($Beta5$), for the smallest stocks ($Size1$), and for stocks with the high book-to-market ratios ($BTM4$ and $BTM5$).

In this study we examine the role of a disinformation aggregate risk by a factor that picks up the differential return between stocks with low analyst coverage (attention) and stocks with high analyst coverage. There are reasons for considering a seasonal pattern in our risk factor. Firstly, the window dressing hypothesis (Haugen and Lakonishok, 1988; Lakonishok *et al.*, 1991) suggests that stocks of small and risky firms are subject to selling pressure at year-end which reverses in January. Therefore, average returns for highly visible (followed) firms will be lower in January as compared to the other months of the year. This would produce a special increase in the attention factor in January. Secondly, given that the earnings estimates provided by analysts are related to the available accounting information for the firm, it would be possible to think that the attention level varies at the time that information is available.[11] Some empirical evidence supports this idea. Bhardwaj and Brooks (1992) studied the neglected firm effect and their findings point out a strong relationship between the January effect and the neglected effect. Zeghal (1984) gives a possible explanation for this issue. He reports that financial statements of firms with low level of information are more informative than statements of enterprises with high levels of information. Therefore, in January, the new information available on firms with low attention during the year could reduce investors' uncertainty about them and their demanded returns. If this was the case, our attention risk factor would be lower in January than for the other months in the year.

In any case, we could think that the results reported in the previous section are due to relationships between returns and attention risk that occur only in particular months. This would cut the confidence of the model tested here. In order to investigate this issue, let us have a look at the risk premiums estimated in the different months.

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[11] This mainly happens at the end of the year in Spain.

TABLE 8
Cross-sectional regressions. January Effect

Panel A: January						
γ_0	γ_1	γ_2	γ_3	γ_4	Adj. R^2 (%)	SSR
0.0052	**0.0556**				41.34	19.456
(0.34)	(2.52)					
[0.27]	[2.01]					
-0.0011	**0.0619**			0.0342	52.89	10.535
(-0.09)	(2.97)			(2.89)		
[-0.06]	[2.00]			[1.95]		
0.0254	0.0351	**0.0421**	0.0017		59.80	4.397
(1.77)	(1.97)	(3.48)	(0.15)			
[0.98]	[1.09]	[1.92]	[0.08]			
0.0215	0.0389	**0.0457**		0.0126	60.54	4.108
(1.26)	(2.17)	(3.29)		(1.07)		
[0.64]	[1.11]	[1.69]		[0.55]		
0.0013	**0.0595**		0.0140	**0.0287**	56.80	7.430
(0.09)	(3.05)		(1.17)	(2.41)		
[0.06]	[2.17]		[0.83]	[1.71]		
0.0176	0.0429	**0.0400**	0.0041	0.0117	59.75	3.344
(1.14)	(2.33)	(3.26)	(0.36)	(1.03)		
[0.65]	[1.33]	[1.86]	[0.20]	[0.59]		
Panel B: Rest of the year						
γ_0	γ_1	γ_2	γ_3	γ_4	Adj. R^2 (%)	SSR
0.0152	-0.0028				22.27	0.7120
(4.84)	(-0.58)					
[4.83]	[-0.58]					
0.0143	-0.0019			0.0051	41.18	0.6888
(4.41)	(-0.37)			(1.08)		
[4.38]	[-0.37]			[1.07]		
0.0132	-0.0007	-0.0010	-0.0019		46.96	0.5605
(3.69)	(-0.13)	(-0.30)	(-0.49)			
[3.68]	[-0.13]	[-0.30]	[-0.49]			
0.0095	0.0030	-0.0040		**0.0087**	49.55	0.5775
(2.73)	(0.52)	(-1.35)		(1.82)		
[2.63]	[0.50]	[-1.30]		[1.75]		
0.0135	-0.0010		-0.0011	0.0046	46.93	0.5650
(4.16)	(-0.20)		(-0.30)	(1.01)		
[4.14]	[-0.20]		[-0.30]	[1.01]		
0.0091	0.0034	-0.0035	0.0013	**0.0074**	51.65	0.4870
(2.61)	(0.61)	(-1.06)	(0.36)	(1.57)		
[2.52]	[0.59]	[-1.03]	[0.35]	[1.52]		

Notes:
1. The table is based on all sample observations (firm-monthly) during the period January 1991- December 2006 where we have data about stocks returns, market values, book-to-market ratios, and number of financial analysts' estimates.
2. This table reports the estimates from the two-step cross-sectional Fama-MacBeth procedure for the following regression, from 1993:12 to 2006:12.

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{mpt} + \gamma_{2t}\hat{\beta}_{SMBpt} + \gamma_{3t}\hat{\beta}_{HMLpt} + \gamma_{4t}\hat{\beta}_{LMMApt} + \upsilon_{pt}$$

The dependent variable is the monthly return on twenty portfolios constructed by sorting stocks into five attention portfolios, five size portfolios, five boot-to-market portfolios, and five market beta portfolios. The explanatory variables are the betas of the different factors, and they are estimated with previous data to each cross-sectional estimation, including the month in question, by the time-series regression.

$$R_{pt} - R_{ft} = \alpha_p + \beta_{pm}(R_{mt} - R_{ft}) + \beta_{SMBp}SMB_t + \beta_{HMLp}HML_t + \beta_{LMMApt}LMMA_t + \varepsilon_{pt}$$

$(R_{mt} - R_{ft})$, SMB and HML are the three Fama-French factors and LMMA is a risk factor constructed as the difference between the return of the less attended stocks minus the return of the more attended ones.
The risk premium estimates are the average of the coefficient estimates from the monthly cross-sectional regressions. t-values for individual significance, calculated using the standard deviation of the series of estimates, are reported in parenthesis. The corresponding adjusted t-statistics by Shanken (1992) are in brackets.
3. Adj. R^2 is the adjusted R-square computed from the R-square obtained with the sum of all total sums and all residual sums from the monthly cross-sectional regressions. SSR is the sum of the square of the residuals for the twenty portfolios.

Table 8 reports the results of the estimation of equation [14] for only January months, on the one hand, and for the rest of the year, on the other. Firstly, the results show that the market beta has heavy seasonal behaviour. As usual, the market premium is positive and highly significant in January. For the rest of the months, this premium is zero, indicating the failure of the CAPM to explain all the returns. With respect to the other two factors by Fama and French, we also observe how the size effect has not disappeared at all. The size risk premium is still high in January, while it is negative and statistically insignificant during the other months. With regards to the attention risk factor, we do not find a seasonal pattern. Its premium estimate and its t-statistic are higher in January than that obtained for the whole sample (Table 5) for models in which SMB is not included. However, when the January effect is controlled by the risk factor based on size, our $LMMA$ factor keeps its moderate significance shown in Table 5, for both January and the rest of the year.

Again, despite of the debatable significance of the attention beta given its t-statistics, the adjusted R^2 increases and the mean square error (SRR) decreases when the attention factor is included, for both the January sample and the rest of the year.

Therefore, we can understand that the attention factor collects risk associated to the quantity of information in the market reasonably well, because it determines a part of asset returns and it occurs in this way during all the months in the year.

4.4   Robustness

Several additional analyses were done in order to check the robustness of the results.[12]

−*The set of tested assets*

We use another two sets of portfolios in the cross-sectional estimation of the risk premiums of the four factors considered here.

1 To be consistent with the way in computing our attention factor, we carry out the cross-sectional analysis using ten residual attention portfolios as the assets to be explained by the models. The results remain the same.

[12] These results are not shown here for brevity, but they are available upon request.

2 Also, we estimate the models using eight portfolios constructed by sorting stocks by attention level, without discounting the effect of firm size. In this case, given that the portfolios present lower dispersion in returns, the attention risk premium is smaller than before but again the goodness of fit of the models increases when LMMA is included.

−The attention risk factor

We use three other proxies for the level of information of each stock with two different ways of computing the risk factors.

1 We measure the level of attention for each firm controlling by size and book-to-market characteristics simultaneously. That is,

$$\log\left(1 + EST_{jt}\right) = \beta_0 + \beta_1 \log\left(SIZE_{jt}\right) + \beta_2 \log\left(BTM_{jt}\right) + \varepsilon_{jt}$$

Then, we construct the $LMMA$ factor sorting stocks by the residual of the regression and placing them into three groups of attention. $LMMA$ is the difference between the return on the group with the lowest attention stocks and the return on the group with the highest attention stocks. Given the construction of this factor, the Fama-French factors are computed in the traditional way.

2 In this other case, we directly use the number of earnings estimates to sort stocks by level of attention. Then, we compute SMB, HML, and LMMA interacting two groups of size, two groups of book-to-market, and two groups of attention.

3 The total sum of monthly number of recommendations and EPS estimates for each firm.

In all cases, the cross-sectional results for the analysis of our twenty portfolio returns remain qualitatively the same.

5. Conclusions

Financial literature has shown certain evidence of a relationship between stock returns and the quantity of available information about the assets.[13] However, except for a few cases, asset pricing models

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[13] See the introduction for examples.

have been developed under complete information. With this in mind, the goal of this study is to analyse whether such a relationship exists.

We propose a model that incorporates a risk premium related to the disinformation in the market, in line with the theoretical model of Merton (1987). Then we test the performance of this model against the traditional CAPM and the Fama and French (1993) model, using Spanish market data from 1991 to 2006.

For the empirical application, we use the attention of financial analysts on each stock as a proxy for the quantity of available information of a firm. Empirical literature suggests that investors demand lower returns for assets with more available information. So, we understand that lack of information can be interpreted as a source of risk. Therefore, we propose the construction of a systematic risk factor based on this proxy of information as the difference between the returns of less analysts following and those of more analysts following. Under these arguments, the disinformation risk factor should present a positive price in the market.

The empirical analysis shows that this is actually the case. We find that both a standard CAPM and the Fama-French model improve when the disinformation factor is included. More precisely, the risk premium for our factor is positive and moderately significant in explaining cross-sectional returns; performance measures for both a time-series analysis and the cross-sectional estimation of the models indicate a relevant contribution of the disinformation risk factor, and lower pricing errors for each analyzed portfolio cross-sectionally when this risk factor is considered in the model. Also we find that the results remain qualitatively the same when we carry out the analysis separating January months from the rest of the year, despite the strong January effect found in the Spanish stock returns. Identical conclusions can be found when we use alternative proxies for the information about the assets, alternative ways of computing the disinformation risk factor, and alternative sets of portfolio returns in the cross-sectional analysis. Therefore, we can be confident about our results.

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Resumen

Generalmente, la teoría de valoración de activos asume que los mercados son perfectos y, en consecuencia, los modelos de valoración no consideran la posibilidad de que existan deficiencias de información durante el proceso de formación de precios de los activos. Nuestro trabajo analiza si la cantidad de información disponible sobre un activo determina, de algún modo, su rentabilidad. Concretamente, pretendemos analizar si existe un factor de riesgo sistemático asociado a la información que haga que aquellos activos muy sensibles a esta fuente de riesgo presenten mayores rentabilidades medias. Nuestros resultados indican que, efectivamente, el mercado valora el riesgo de desinformación. Encontramos mejoras sustanciales en el ajuste de modelos tradicionales como el CAPM o el modelo de Fama y French cuando incorporamos este factor de riesgo, tanto en el análisis de serie temporal como en el de sección cruzada.

Palabras clave: Valoración de activos, información incompleta, estimación de beneficios por parte de analistas financieros.